THREE-DIMENSIONAL SUPERSONIC FLOWS AT LARGE DISTANCES FROM A BODY OF FINITE VOLUME

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The equations of gasdymanics, transferred to independent variables consisting of the pressure and two stream functions, are simplified under the assumption that the zone of perturbed motion is narrow and variations of the flow parameters are small. In physical space simplifications of this kind are usually applied to the description of the "shortwave" type of flows [1-3]. We construct the general solution of the approximate equations in a form suitable for studying the perturbed flow at sufficiently large distances in the flow over a body of three-dimensional configuration. We show that there exist planes in each of which the motion may be described quasi-two-dimensionally by means of relations for flows with axial symmetry. We study the influence of stream surface curvature on the asymptotic state of the motion. We examine limiting transitions of axially symmetric flows to the flows in question [4].

1. We consider the system of equations of three-dimensional stationary flows of a nonviscous and nonheat-conducting gas with arbitrary thermodynamic properties. We assume that the equation of state is given in the form of a dependence of the heat content h on the pressure p, and the entropy s. The continuity equation is satisfied identically if the mass-flow density vector is written in the form of the vector product of the gradients of two scalar functions ψ and φ

$$\rho \mathbf{v} = \nabla\psi \times \nabla\varphi \tag{1.1}$$

The functions ψ and φ are three-dimensional analogs of the stream functions of two-dimensional flows; they are constant along the stream lines

$$(\nabla\psi \cdot \mathbf{v}) = 0, \quad (\nabla\varphi \cdot \mathbf{v}) = 0$$

the product of their differentials gives the mass flow in a stream tube.

We consider p, ψ, and φ as the independent variables, and the cartesian coordinates x_i as the unknown functions. Then, from the equations of motion, of energy, and the relations (1.1), we obtain three partial differential equations for the functions $x_i(p, \psi, \varphi)$ [5]

$$\frac{\partial}{\partial p}\left[\frac{w\,\partial x_i / \partial p}{\sqrt{(\partial x_k / \partial p)(\partial x_k / \partial p)}}\right] = -\frac{\partial (x_j, x_k)}{\partial (\psi, \varphi)} \tag{1.2}$$

where $w = \sqrt{2(h_m - h)}$, (i, j, k) is subject to the cyclic substitution (1, 2, 3), and h_m is the deceleration enthalpy.

The velocity vector components may be obtained from the following relations

$$v_i = \frac{w\,\partial x_i / \partial p}{\sqrt{(\partial x_k / \partial p)(\partial x_k / \partial p)}} \qquad (i = 1, 2, 3) \tag{1.3}$$

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Putting $x_3/x_2 = \tan\varphi$ and assuming that the Cartesian coordinates, and all the gasdynamic quantities are independent of φ, we obtain the equations of axially symmetric flows [6] from Eqs. (1.2), and when $x_3 = \varphi$, the equations of two-dimensional flows.

2. At large distances from the body, the perturbed flow region consists of a comparatively narrow zone in the neighborhood of some surface $x_i = u_i(\psi, \varphi)$. We seek the functions $x_i(p, \psi, \varphi)$ in the form

$$x_i = u_i(\psi, \varphi) + \xi_i^\circ \xi_i(p, \psi, \varphi) \tag{2.1}$$

where the ξ_i° are constants giving the orders of the perturbations of the streamlines. We consider weak shock waves; therefore, as the surfaces in the neighborhood of which we seek a solution, we take the characteristic surface of the unperturbed flow, the equation for which has the following form in the p, ψ, and φ variables:

$$d_1^2 + d_2^2 + d_3^2 + \left(\frac{\partial^2 h}{\partial p^2}\right)_\infty = 0, \quad d_i = \frac{\partial(u_j, u_k)}{\partial(\psi, \varphi)} \tag{2.2}$$

where (i, j, k) is the cyclic substitution (1, 2, 3), and the subscript ∞ denotes quantities defined with respect to parameters of the oncoming flow. The velocity vector of the oncoming flow is directed along the x_1 axis; therefore

$$d_1 = \frac{\partial(u_2, u_3)}{\partial(\psi, \varphi)} = -\left(\frac{\partial w}{\partial p}\right)_\infty$$

The velocity vector of the perturbed flow is defined by the two angles α and β

$$v_1 = w\cos\beta\cos\alpha, \quad v_2 = w\cos\beta\sin\alpha, \quad v_3 = w\sin\beta \tag{2.3}$$

Perturbations of the flow parameters are small; therefore

$$\alpha \ll 1, \quad \beta \ll 1, \quad \varepsilon = (p - p_\infty)/p_\infty \ll 1$$

We make the following substitutions:

$$\alpha \to \alpha^\circ\alpha, \quad \beta \to \beta^\circ\beta, \quad \varepsilon \to \varepsilon^\circ\varepsilon$$

and let the constants α°, β°, and ε° give the orders of the corresponding perturbations. We assume that the directions of the x_2 and x_3 axes in the flow are equivalent, therefore, we put $\alpha^\circ = \beta^\circ$.

At the front of the weak shock wave the flow turn angle is

$$\vartheta_1 = \sqrt{\left(\frac{\partial^2 w}{\partial p^2}\right)_\infty \frac{p_\infty^2}{w_\infty}\,\frac{p - p_\infty}{p_\infty}}$$

From this, noting that $\vartheta_1 = \sqrt{\alpha^2 + \beta^2}\,\alpha^\circ$, we obtain

$$\alpha^\circ = \beta^\circ = \varepsilon^\circ \tag{2.4}$$

We expand the quantities (2.3) in series in powers of α° and β°, and the modulus of the velocity in a series in Δp and Δs, noting that the entropy increment at the jump is $\Delta s \sim \Delta p^3$. If, now we substitute the expressions for the unknown functions into Eqs. (1.2), we obtain

$$\begin{gathered}
\left[\left(\frac{\partial^2 w}{\partial p^2}p\right)_\infty \varepsilon - w_\infty \frac{\partial}{\partial\varepsilon}\left(\frac{\alpha^2+\beta^2}{2p_\infty}\right)\right]\varepsilon^\circ + \left[\left(\frac{\partial^3 w}{\partial p^3}p^2\right)_\infty \frac{\varepsilon^2}{2} - \left(\frac{\partial w}{\partial p}\right)_\infty \frac{\partial}{\partial\varepsilon}\left(\varepsilon\frac{\alpha^2+\beta^2}{2}\right)\right]\varepsilon^{\circ 2} = \\
= -\left(\frac{\partial w}{\partial p}\right)_\infty - \frac{\partial(u_2, u_3)}{\partial(\psi, \varphi)} - \xi_2^\circ\frac{\partial(\xi_2, u_3)}{\partial(\psi, \varphi)} - \xi_3^\circ\frac{\partial(u_2, \xi_3)}{\partial(\psi, \varphi)}. \\
\frac{w_\infty}{p_\infty}\frac{\partial\alpha}{\partial\varepsilon} + \alpha^\circ\left(\frac{\partial w}{\partial p}\right)_\infty\frac{\partial(\varepsilon\alpha)}{\partial\varepsilon} = -\frac{\partial(u_3, u_1)}{\partial(\psi, \varphi)} - \xi_1^\circ\frac{\partial(u_3, \xi_1)}{\partial(\psi, \varphi)} - \xi_2^\circ\frac{\partial(\xi_3, u_1)}{\partial(\psi, \varphi)}, \\
\frac{w_\infty}{p_\infty}\frac{\partial\beta}{\partial\varepsilon} + \beta^\circ\left(\frac{\partial w}{\partial p}\right)_\infty\frac{\partial(\beta\varepsilon)}{\partial\varepsilon} = -\frac{\partial(u_1, u_2)}{\partial(\psi, \varphi)} - \xi_1^\circ\frac{\partial(\xi_1, u_2)}{\partial(\psi, \varphi)} - \xi_2^\circ\frac{\partial(u_1, \xi_2)}{\partial(\psi, \varphi)} \\
\frac{\alpha^\circ\xi_1^\circ}{\varepsilon^\circ}\alpha\frac{\partial\xi_1}{\partial\varepsilon} = \frac{\xi_2^\circ}{\varepsilon^\circ}\frac{\partial\xi_2}{\partial\varepsilon}, \quad \frac{\beta^\circ\xi_1^\circ}{\varepsilon^\circ}\beta\frac{\partial\xi_1}{\partial\varepsilon} = \frac{\xi_3^\circ}{\varepsilon^\circ}\frac{\partial\xi_3}{\partial\varepsilon}
\end{gathered} \tag{2.5}$$

The last two equations furnish a description of the streamlines in terms of the variables p, ψ, and φ

$$\frac{\partial x_1}{v_1} = \frac{\partial x_2}{v_2} = \frac{\partial x_3}{v_3}$$

From this, it follows that

$$\xi_1^\circ = \frac{\xi_2^\circ}{\alpha^\circ} = \frac{\xi_3^\circ}{\beta^\circ} \tag{2.6}$$

Integrating the second and third of Eqs. (2.5) to within small ε° and ξ_1°, we obtain

$$\alpha = \frac{p_\infty}{w_\infty} \frac{\partial (\xi_*, u_3)}{\partial (\psi, \varphi)} + \alpha_1 (\psi, \varphi), \quad \beta = \frac{p_\infty}{w_\infty} \frac{\partial (u_2, \xi_*)}{\partial (\psi, \varphi)} + \beta_1 (\psi, \varphi) \tag{2.7}$$

where

$$\xi_* = u_1 \varepsilon - \varepsilon^\circ \left(\frac{\partial w}{\partial p} \frac{p}{w}\right)_\infty \varepsilon^2 u_1 + \xi_1^\circ \int \xi_1 d\varepsilon$$

For the uniform oncoming flow, based on conservation of the tangential velocity components, it follows that

$$\alpha_1 (\psi, \varphi) \equiv \beta_1 (\psi, \varphi) \equiv 0$$

Substituting α and β into the first of Eqs. (2.5), and using Eqs. (2.2) and (2.6), we obtain $\xi_1^\circ = \varepsilon^\circ$ and the equation for $\xi_1(\varepsilon, \psi, \varphi)$

$$\frac{\partial (u_1, u_2)}{\partial (\psi, \varphi)} \frac{\partial (\xi_1, u_2)}{\partial (\psi, \varphi)} + \frac{\partial (u_1, u_3)}{\partial (\psi, \varphi)} \frac{\partial (\xi_1, u_3)}{\partial (\psi, \varphi)} = -\frac{p_\infty}{2} \left(\frac{\partial^3 h}{\partial p^3}\right)_\infty \varepsilon + \frac{\varepsilon}{2} \frac{\partial \xi_1}{\partial \varepsilon} \left[\frac{\partial (d_3, u_2)}{\partial (\psi, \varphi)} - \frac{\partial (d_2, u_3)}{\partial (\psi, \varphi)}\right] \tag{2.8}$$

As the independent variables of the problem it is convenient to use the variables u_2 and u_3,

$$\frac{\partial (u_2, u_3)}{\partial (\psi, \varphi)} = -\left(\frac{\partial w}{\partial p}\right)_\infty \neq 0$$

these variables represent the Cartesian coordinates of the points of intersection of the streamlines with the characteristic surface (2.2). After the transition to u_2 and u_3 in Eqs. (2.7) and (2.8), we obtain a system of equations for the functions $\xi_i(\varepsilon, u_2, u_3)$, wherein α and β are to be obtained from Eqs. (2.7), and terms of the first order only are to be retained. We note also that to obtain perturbations of the coordinate x_1 to within quantities of the first order, use of an equation of state to within quantities of the third order would be required, however, the perturbed flow in this approximation would remain irrotational.

3. In the plane of the variables u_2 and u_3, we change over to polar coordinates $u_2 = r \cos \theta$, $u_3 = r \sin \theta$, and we write the equation of the characteristics (2.2) in these coordinates:

$$\left(\frac{\partial u_1}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u_1}{\partial \theta}\right)^2 = \frac{w_\infty^3 (\partial^2 w / \partial p^2)_\infty}{(\partial h / \partial p)_\infty} = M_\infty^2 - 1 \tag{3.1}$$

Equation (3.1) may be satisfied identically, if we introduce a variable η through the relations

$$\frac{\partial u_1}{\partial r} = \sqrt{M_\infty^2 - 1} \cos \eta, \quad \frac{\partial u_1}{d\theta} = \sqrt{M_\infty^2 - 1}\, r \sin \eta \tag{3.2}$$

From this it follows that

$$r \sin \eta = f(\theta + \eta) \tag{3.3}$$

where $f(\theta + \eta)$ is an arbitrary function.

If as independent variables, we select r and $\lambda = \theta + \eta$, the solution of Eq. (3.1) then has the following form:

$$u_1 (r, \lambda) = \sqrt{M_\infty^2 - 1} \left[\sqrt{r^2 - f^2(\lambda)} + \int_0^\lambda f(\lambda') d\lambda'\right] \tag{3.4}$$

We rewrite the equations for ξ_1, ξ_2, ξ_3 in the variables ε, r, and λ:

$$\frac{\sqrt{r^2 - f^2(\lambda)}}{r}\frac{\partial \xi_1}{\partial r} - \frac{\varepsilon}{2\sqrt{r^2 - f^2(\lambda)} - 2f'(\lambda)}\frac{\partial \xi_1}{\partial \varepsilon} = \mu\varepsilon \tag{3.5}$$

$$\frac{\partial \xi_2}{\partial \varepsilon} = \sqrt{\left(\frac{\partial^2 w}{\partial p^2}\right)_\infty \frac{p_\infty^2}{w_\infty}}\, \varepsilon \cos\lambda \frac{\partial \xi_1}{\partial \varepsilon}$$

$$\frac{\partial \xi_3}{\partial \varepsilon} = \sqrt{\left(\frac{\partial^2 w}{\partial p^2}\right)_\infty \frac{p_\infty^2}{w_\infty}}\, \varepsilon \sin\lambda \frac{\partial \xi_1}{\partial \varepsilon}$$

Here

$$\mu = -\frac{w_\infty^2 p_\infty (\partial^3 h / \partial p^3)_\infty}{2\sqrt{M_\infty^2 - 1}\,(\partial h / \partial p)_\infty}$$

For a polytropic gas

$$\mu = -\frac{(k+1) M_\infty^2}{2k\sqrt{M_\infty^2 - 1}}, \qquad \sqrt{\left(\frac{\partial^2 w}{\partial p^2}\right)_\infty \frac{p_\infty^2}{w_\infty}} = \frac{\sqrt{M_\infty^2 - 1}}{kM_\infty^2}$$

We integrate the system of equations (3.5) and, taking Eq. (3.4) into account, we write the solution for the x_i in the following form:

$$x_1 = X(\gamma, \sigma, \tau) + \sqrt{M_\infty^2 - 1}\left[\int_0^\lambda f(\lambda')\,d\lambda' + f'(\lambda)\right] \tag{3.6}$$

$$x_2 = Y(\gamma, \sigma, \tau)\cos\lambda + f'(\lambda)\cos\lambda + f(\lambda)\sin\lambda$$

$$x_2 \sin\lambda - x_3\cos\lambda = f(\lambda)$$

$$\gamma = \sqrt{r^2 - f^2(\lambda)} - f'(\lambda), \qquad \sigma = \varepsilon\sqrt{\gamma}$$

$$X(\gamma, \sigma, \tau) = \sqrt{M_\infty^2 - 1}\,\gamma + 2\mu\sigma\sqrt{\gamma} + \tau(\lambda, \sigma)$$

$$Y(\gamma, \sigma, \tau) = \gamma + \frac{\sqrt{M_\infty^2 - 1}}{kM_\infty^2}\left[\mu\sigma^2 + \gamma^{-1/2}\int_0^\sigma \frac{\partial \tau}{\partial \sigma'}\sigma' d\sigma'\right]$$

where $\tau(\lambda, \sigma)$ is an arbitrary function.

In Eqs. (3.6), we have used the fact that when $\varepsilon = 0$ the perturbations of the coordinates ξ_i are equal to zero.

4. Let us assume that on the stream surface S_0, given by the equation $R = R(x, \vartheta)$ (R, x, and ϑ are cylindrical coordinates), the distribution of pressure is known to be $\varepsilon = \varepsilon_0(x, \vartheta)$. The family of characteristics of the system of Eqs. (3.5)

$$\varepsilon\left[\sqrt{r^2 - f^2(\lambda)} - f'(\lambda)\right]^{1/2} = \sigma = \text{const} \tag{4.1}$$

includes the characteristic surface of the unperturbed flow ($\varepsilon = 0$, $\sigma = 0$), which consists of the envelope of the family of Mach cones passing through the curve L, on which

$$\varepsilon_0(x_l, \vartheta_l) = 0, \quad R_l = R(x_l, \vartheta_l) \tag{4.2}$$

Since on L, we have

$$R_l = r, \quad x_l = u_1, \quad \vartheta_l = \theta$$

then, from Eqs. (4.2), we can find $r = R_l(\theta_l)$ and $u_1 = u_1(\theta_l)$, wherein

$$\frac{dx_l}{d\theta_l} = \frac{\partial u_1}{\partial \theta} + \frac{\partial u_1}{\partial r}\frac{dR_l}{d\theta_l} \tag{4.3}$$

where

$$\frac{dx_l}{d\theta_l} = -\frac{\partial\varepsilon_0}{\partial\theta_l}\left[\frac{\partial\varepsilon_0}{\partial x_l}\right]^{-1}, \quad \frac{dR_l}{d\theta_l} = \frac{\partial(R, \varepsilon_0)}{\partial(\theta_l, x_l)}\left[\frac{\partial\varepsilon_0}{\partial x_l}\right]^{-1}$$

From this, using Eqs. (3.2) and (3.3), we obtain $f(\lambda)$ in parametric form

$$f(\theta_l) = \left\{\frac{dx_l}{d\theta_l}\frac{1}{\sqrt{M_\infty^2 - 1}} - \frac{dR_l}{d\theta_l}\sqrt{1 + \frac{1}{R_l^2}\left[\left(\frac{dx_l}{d\theta_l}\right)^2\frac{1}{M_\infty^2-1} - \left(\frac{dR_l}{d\theta_l}\right)^2\right]}\right\} \cdot \left[1 + \frac{1}{R_l^2}\left(\frac{dR_l}{d\theta_l}\right)^2\right]^{-1}$$

$$\lambda(\theta_l) = \theta_l + \arcsin\frac{f(\theta_l)}{R_l(\theta_l)} \tag{4.4}$$

In the general solution (3.6), instead of $\tau(\lambda, \sigma)$, it is more convenient to use the inverse function $\sigma(\lambda, \tau)$, which, in contrast to $\tau(\lambda, \sigma)$, is a single-valued function of its arguments by virtue of its proportionality to the pressure. The relationship between the parameters r and λ on the curve $L(r = R_l[\theta_l(\lambda)])$ is preserved along a streamline, therefore, on the surface S_0, the equation $\gamma = \gamma(\lambda, r(\lambda))$ is satisfied. Taking this fact into account, we obtain the following equation for the function $\sigma(\lambda, \tau)$

$$\sigma(\lambda, \tau) = \sqrt{\gamma(\lambda)}\,\varepsilon_0[x_1(\lambda, \sigma, \tau), \vartheta(\lambda, \sigma, \tau)] \tag{4.5}$$

If in some domain of values of τ the derivative $d\sigma/d\tau > 0$, intersection of the characteristics becomes a possibility, and thence, the development of a shock wave. If we denote the parameters of two intersecting characteristics by τ_1 and τ_2, $\tau_1 < \tau_2$, then, at the point of intersection $\xi_{i1} = \xi_{i2}$, and from this, we obtain equations relating the flow parameters on the two sides of the jump

$$2\mu[\sigma(\lambda, \tau_1) - \sigma(\lambda, \tau_2)]\sqrt{\gamma} = \tau_2 - \tau_1 \tag{4.6}$$

$$\mu[\sigma^2(\lambda, \tau_1) - \sigma^2(\lambda, \tau_2)]\sqrt{\gamma} = \int_{\tau_1}^{\tau_2}\sigma(\lambda, \tau)\,d\tau$$

From the relations (3.6) and (4.6), we can obtain the following equation for the angle of inclination of the jump:

$$\operatorname{ctg}\omega = [M_\infty^2 - 1]^{1/2} + {}^1/_2\mu(\varepsilon_1 + \varepsilon_2)$$

coinciding with the exact expression for cot ω to within quantities of the first order.

5. When $f(\lambda) = 0$ the solution (3.6) given above reduces to that for the axially symmetric case. In addition,

$$\lambda = \theta, \gamma = r, \varepsilon\sqrt{r} = \sigma(\tau), x_1 = X(r, \sigma, \tau), x_2 = Y(r, \sigma, \tau)$$

and from this solution, we can then obtain the results given in [4, 2]. When the parameter λ is constant, the general solution (3.6) for the three-dimensional case depends on the variables γ and σ in exactly the same way (to within constants) that the axially symmetric solution depends on r and σ. The surfaces λ = const are planes parallel to the x_1 axis, intersecting the x_3 axis at the point $-f(\lambda)/\cos\lambda$, and the angle of inclination of a λ plane to the x_2 axis is equal to λ. The velocity vector lies in the λ plane $\beta = \alpha\tan\lambda$. Flows in the λ planes are independent, and the pressure asymmetry in λ, created on the initial stream surface, is maintained with an increase in the distance. This result is a consequence of the fact that the zone of the perturbed flow is relatively narrow, the pressure gradients in directions tangent to the surface of the front are small and, in the resulting (first) approximation, may be discounted. In each λ plane, the shock waves decay differently, even when the initial perturbations are identical, since the parameter γ, which plays the role of a distance, depends on λ. As the distance from the axis increases,

$$\lambda - \theta = \arcsin f(\lambda)/r \to 0, \quad \gamma - r \to f'(\lambda) \ll r$$

i.e., the geometrical flow parameters tend to those for axial symmetry.

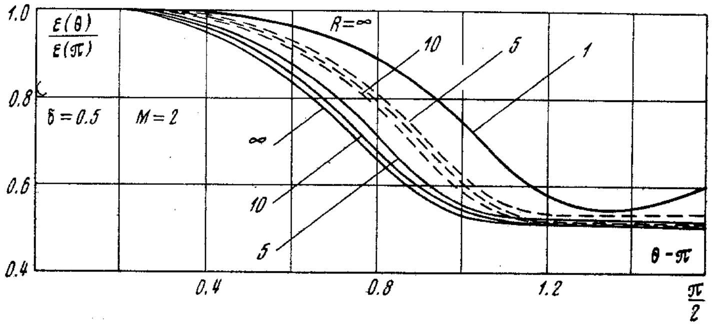

Fig. 1

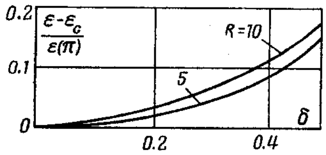

Fig. 2

We consider the effect of the curvature of the initial surface on the magnitude of the perturbations in a λ plane below the body. By virtue of symmetry on the initial surface

$$\frac{dx_l}{d\theta_l} = \frac{dR_l}{d\theta_l} = 0 \quad \text{for} \quad \theta_l = \pi$$

From this it follows that

$$\lambda = \pi, \ f(\pi) = 0$$

If, when $\lambda = \pi$, we assume that the pressure distribution $\varepsilon_0(\pi, \tau)$ is known, then, in the plane $\lambda = \pi$ the pressure depends on the distance in the following way:

$$\varepsilon(\pi, r, \tau) = \varepsilon_0(\pi, \tau)\,[R_l(\pi)]^{1/2}\left[r + \left(\frac{r}{R_l(\pi)} - 1\right)\frac{df}{d\theta_l}(\pi)\right]^{-1} \tag{5.1}$$

where

$$\frac{df}{d\theta_l} = \frac{d^2}{d\theta_l^2}\left(\frac{x_l}{\sqrt{M_\infty^2 - 1}} - R_l\right)$$

From this, we see that for the pressure below the body to decrease, it is necessary that the derivative $df/d\theta_l$ be positive and possibly large. The configuration of a curve L with such a derivative arises, for example, on a sufficiently narrow swept-back wing. An increase in the sweep-back angle leads to a decrease in the perturbations at all distances below the initial surface since a change in the curvature of the zero pressure curve leads to a change in the function $\sigma(\lambda, \tau)$. From Eq. (5.1), it also follows that an increase in the curvature $k = 1 - R_l''/R_l$ of the curve L in the plane x_1 = const gives rise to a decrease in the pressure in the plane of symmetry. Such an increase in the curvature of the curve L may be attained by an increase in the transverse V-angle of the wing in the flow. These results were confirmed experimentally in [7], where a study was made of the effect of wing shape on the sonic boom below the aircraft.

To obtain the flow fields at large distances from bodies of complicated three-dimensional form or from axially symmetric bodies to which the linear theory is not applicable, we can determine the pressure field at distances on the order of several body lengths from the axis in wind tunnel tests and then rescale these measurements for large distances. We give an example of this type of conversion. Assume that we have an initial surface S_0 on which the zero excess pressure curve has the form of an ellipse in the plane x_1 = const. We assume that in the planes θ_l = const the pressure is a linear function of the deviation from the characteristic $\sigma = 0$, and that the pressure gradient $\Delta < 0$ does not depend on θ_l. The pressure drop at the bow shock wave has a maximum $\varepsilon_0(\pi)$ for $\theta_l = 0$ and π, and it decreases to $\varepsilon_0(\pi)/2$ for $\theta_l = \pi/2$ and $3\pi/2$. Figure 1 shows the relative change in the pressure $\varepsilon(\theta)/\varepsilon(\pi)$ in the quadrant $(\pi, 3\pi/2)$ at various distances from the axis (eccentricity of the ellipse is $\delta = 0.5$, linear measurements are with reference to the major axis of the ellipse). The dashed curves denote curves of the pressure $\varepsilon_c(\theta)/\varepsilon(\pi)$, obtained without taking into account the difference of the λ planes from the meridional planes at the initial surface [8]. The essential difference between $\varepsilon(\theta)$ and $\varepsilon_c(\theta)$ is maintained for $R \to \infty$. The difference between the curves when $\theta_l = 3\pi/2$ may be explained by the fact that the curvature of the curve L is larger when $\theta_l = 3\pi/2$ than when $\theta_l = \pi$, and therefore, the damping in the plane $\theta_l = 3\pi/2$ is stronger. Figure 2 presents a graph of the difference $(\varepsilon - \varepsilon_c)/\varepsilon(\pi)$ for $\theta = 3\pi/4$ versus the eccentricity of the ellipse, δ. It is evident that as δ increases, this difference increases rapidly.

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